# Mixed Exercise 2 – Quadratics

1 Solve the following equations without a calculator. Leave your answers in surd form where necessary.

**a** $y^2 + 3y + 2 = 0$ **b** $3x^2 + 13x - 10 = 0$ **c** $5x^2 - 10x = 4x + 3$ **d** $(2x - 5)^2 = 7$

2 Sketch graphs of the following equations:

**a** $y = x^2 + 5x + 4$ **b** $y = 2x^2 + x - 3$ **c** $y = 6 - 10x - 4x^2$ **d** $y = 15x - 2x^2$

E 3 $f(x) = x^2 + 3x - 5$ and $g(x) = 4x + k$, where $k$ is a constant.

**a** Given that $f(3) = g(3)$, find the value of $k$. **(3 marks)**

**b** Find the values of $x$ for which $f(x) = g(x)$. **(3 marks)**

4 Solve the following equations, giving your answers correct to 3 significant figures:

**a** $k^2 + 11k - 1 = 0$ **b** $2t^2 - 5t + 1 = 0$ **c** $10 - x - x^2 = 7$ **d** $(3x - 1)^2 = 3 - x^2$

5 Write each of these expressions in the form $p(x + q)^2 + r$, where $p$, $q$ and $r$ are constants to be found:

**a** $x^2 + 12x - 9$ **b** $5x^2 - 40x + 13$ **c** $8x - 2x^2$ **d** $3x^2 - (x + 1)^2$

E 6 Find the value $k$ for which the equation $5x^2 - 2x + k = 0$ has exactly one solution. **(2 marks)**

E 7 Given that for all values of $x$:

$$3x^2 + 12x + 5 = p(x + q)^2 + r$$

**a** find the values of $p$, $q$ and $r$. **(3 marks)**

**b** Hence solve the equation $3x^2 + 12x + 5 = 0$. **(2 marks)**

E/P 8 The function f is defined as $f(x) = 2^{2x} - 20(2^x) + 64,\ x \in \mathbb{R}$.

**a** Write $f(x)$ in the form $(2^x - a)(2^x - b)$, where $a$ and $b$ are real constants. **(2 marks)**

**b** Hence find the two roots of $f(x)$. **(2 marks)**

9 Find, as surds, the roots of the equation:

$$2(x + 1)(x - 4) - (x - 2)^2 = 0.$$

10 Use algebra to solve $(x - 1)(x + 2) = 18$.

E/P 11 A diver launches herself off a springboard. The height of the diver, in metres, above the pool $t$ seconds after launch can be modelled by the following function:

$$h(t) = 5t - 10t^2 + 10,\ t \geqslant 0$$

**a** How high is the springboard above the water? **(1 mark)**

**b** Use the model to find the time at which the diver hits the water. **(3 marks)**

**c** Rearrange $h(t)$ into the form $A - B(t - C)^2$ and give the values of the constants $A$, $B$ and $C$. **(3 marks)**

**d** Using your answer to part **c** or otherwise, find the maximum height of the diver, and the time at which this maximum height is reached. **(2 marks)**

E/P **12** For this question, $f(x) = 4kx^2 + (4k + 2)x + 1$, where $k$ is a real constant.

**a** Find, in simplest form in terms of $k$, the discriminant of $f(x)$. **(2 marks)**

**b** By simplifying your answer to part **a** or otherwise, prove that $f(x)$ has two distinct real roots for all non-zero values of $k$. **(2 marks)**

**c** Explain why $f(x)$ cannot have two distinct real roots when $k = 0$. **(1 mark)**

E/P **13** Using algebra and showing each stage of your working, find all real solutions of the equation

**i** $2x + \sqrt{x} - 6 = 0$

**ii** $x^8 - 17x^4 + 16 = 0$ **(7 marks)**

E/P **14** Lynn is selling cushions as part of an enterprise project. On her first attempt, she sold 80 cushions at the cost of £15 each. She hopes to sell more cushions next time. Her adviser suggests that she can expect to sell 10 more cushions for every £1 that she lowers the price.

**a** The number of cushions sold $c$ can be modelled by the equation $c = 230 - Hp$, where £$p$ is the price of each cushion and $H$ is a constant. Determine the value of $H$. **(1 mark)**

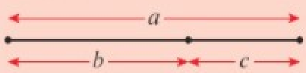
To model her total revenue, £$r$, Lynn multiplies the number of cushions sold by the price of each cushion. She writes this as $r = p(230 - Hp)$.

**b** Rearrange $r$ into the form $A - B(p - C)^2$, where $A$, $B$ and $C$ are constants to be found. **(3 marks)**

**c** Using your answer to part **b** or otherwise, show that Lynn can increase her revenue by £122.50 through lowering her prices, and state the optimum selling price of a cushion. **(2 marks)**

**Challenge**

**a** The ratio of the lengths $a : b$ in this line is the same as the ratio of the lengths $b : c$.




Show that this ratio is $\frac{1 + \sqrt{5}}{2} : 1$.

**b** Show also that the infinite square root

$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}} = \frac{1 + \sqrt{5}}{2}$$